

Gamma-ray Luminosity and Death Lines of Pulsars with Outer Gaps

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ABSTRACT

We re-examine the outer gap size by taking the geometry of dipole magnetic field into account. Furthermore, we also consider that instead of taking the gap size at half of the light cylinder radius to represent the entire outer gap it is more appropriate to average the entire outer gap size over the distance. When these two factors are considered, the derived outer gap size $f(P, B, \langle r \rangle (\alpha))$ is not only the function of period (P) and magnetic field (B) of the neutron star, but also the function of the average radial distance to the neutron star $\langle r \rangle$; which depends on the magnetic inclination angle (α). We use this new outer gap model to study γ -ray luminosity of pulsars, which is given by $L_\gamma = f^3(P, B, \langle r \rangle (\alpha)) L_{sd}$ and L_{sd} is the pulsar spin-down power, as well as the death lines of γ -ray emission of the pulsars. Our model can predict the γ -ray luminosity of individual pulsar if its P, B and α are known. Since different pulsars have different α , this explains why some γ -ray pulsars have very similar P and B but have very different γ -ray luminosities. In determining the death line of γ -ray pulsars, we have used a new criterion based on concrete physical reason, i.e. the fractional size of outer gap at the null charge surface for a given pulsar cannot be larger than unity. In estimate of the fractional size of the outer gap, two possible X-ray fields are considered: (i) X-rays are produced by the neutron star cooling and polar cap heating, and (ii) X-rays are produced by the bombardment of the relativistic particles from the outer gap on the stellar surface (the outer gap is called as a self-sustained outer gap). Since it is very difficult to measure α in general, we use a Monte Carlo method to simulate the properties of γ -ray pulsars in our galaxy. We find that this new outer gap model predicts many more weak γ -ray pulsars, which have typical age between 0.3-3 million years old, than the old model. For all simulated γ -ray pulsars with self-sustained outer gaps, γ -ray luminosity L_γ satisfies $L_\gamma \propto L_{sd}^\delta$; where the value of δ depends on the sensitivity of the γ -ray detector. For the EGRET, δ is ~ 0.38 whereas δ is ~ 0.46 for the GLAST. For γ -ray pulsars with $L_{sd} \lesssim L_{sd}^{crit}$, δ is ~ 1 . $L_{sd}^{crit} = 1.5 \times 10^{34} P^{1/3} \text{ergs}^{-1}$ is determined by $f(\langle r \rangle \sim r_L) = 1$. These results are roughly consistent with the observed luminosity of γ -ray pulsars. These predictions are very different from those predicted by previous outer gap model, which predicts a very flat relation between L_γ and L_{sd} .

Subject headings: pulsars: general-star: neutron- gamma-rays: stars

1. Introduction

High-energy emission models for rotation-powered pulsars are generally divided into polar gap and outer gap models. In polar gap mod-

els, charged particles are accelerated in charge-depleted zones near the pulsar's polar cap and γ -rays are produced through curvature-radiation induced γ -B pair cascade (e.g. Harding 1981; Daugherty & Harding 1996; Zhang & Harding

2000) or through Compton-induced pair cascades (Dermer & Sturmer 1994). In the outer gap models, it is generally accepted that a magnetosphere of charge density

$$\rho_0 \approx \frac{\mathbf{\Omega} \cdot \mathbf{B}}{2\pi c} \quad (1)$$

surrounds a rotating neutron star with magnetic field B and angular velocity Ω (Goldreich & Julian 1969). The magnetospheric plasma is corotating with the neutron star within the light cylinder, at which the corotating speed equals the velocity of light and the distance from the spin axis is

$R_L = c/\Omega$. In the corotating magnetosphere, the electric field along the magnetic field, $E_{||} = \mathbf{E} \cdot \mathbf{B}/B$, is nearly zero. However, the flows of the plasma along open field lines will result in some plasma void regions (where the charge density is different significantly from ρ_0) in the vicinity of null charge surfaces where $\mathbf{\Omega} \cdot \mathbf{B} = 0$ (Holloway 1973). In such charge deficient regions, which are called outer gaps, $E_{||} \neq 0$ is sustained, electrons/positrons can be accelerated to relativistic energies and the subsequent high-energy gamma-ray emission and photon-photon pair production can maintain the current flow in the magnetosphere (Cheng, Ruderman & Sutherland 1976; Cheng, Ho & Ruderman 1986a, 1986b, hereafter CHRI and CHR II; Romani, 1996; Zhang & Cheng 1997; Hirotani 2001).

Based on known γ -ray pulsars, the luminosity and conversion efficiency of γ -rays in various models have been studied (for example, Harding 1981; Dermer & Sturmer 1994; Rudak & Dyks 1998; Yadigaroglu & Romani 1995; Zhang & Cheng 1998). Observations by Compton Gamma-ray Observatory (CGRO) show that γ -ray luminosity of rotation-powered pulsars is proportional to square root of the spin-down power (Thompson et al. 2001). Using the recent new polar gap models (e.g. Zhang & Harding 2000; Harding & Muslimov 2001; Harding, Muslimov & Zhang, 2002), Harding et al. (2002) have studied the decline of γ -ray pulsars based on the predicted luminosity of γ -ray pulsars $L_\gamma \propto L_{sd}^\delta$, where $\delta \sim 0.5$ when $L_{sd} \gtrsim L_{sd}^{break}$ and $\delta \sim 1$ when $L_{sd} \lesssim L_{sd}^{break}$ respectively and $L_{sd}^{break} = 5 \times 10^{33} P^{-1/2}$ erg/s.

For a rapid rotating pulsar, it is believed that its spin-down power, L_{sd} , is converted into radiation energy. Because the outer gap occupies only a

part of the open field line region, the gamma-ray luminosity produced in the outer gap is a fraction of the spin-down power. It has been shown that the gamma-ray luminosity in the outer gap is proportional to f^3 , i.e. $L_\gamma \approx f^3 L_{sd}$ (CHR II; Zhang & Cheng 1997). In previous works, the fractional size of outer gap only depends on the period and magnetic field for a gamma-ray pulsar. For example, the fractional size of the outer gap for Crab-like pulsars is $f \propto B_{12}^{-13/20} P^{33/20}$ (CHR II). In the outer gap model described by Zhang & Cheng (1997), $f \propto B^{-4/7} P^{26/21}$, the observed gamma-ray luminosities with energies greater than 100 MeV from the known gamma-ray pulsars except for the Crab pulsar can be explained approximately (Zhang & Cheng 1998). It should be noted that the model predicts that the γ -ray pulsars with the same values of $(B/P)^{0.3}$ have same γ -ray luminosities. For example, the ratio of $(B/P)^{0.3}$ for PSR B1055-52 to that for Geminga is ~ 0.9 , it means that the ratio of both gamma-ray luminosities is ~ 0.9 . However, the observed ratio of the luminosities with energies greater than 100 MeV of these two pulsars are ~ 8 (Kaspi et al. 2000). Briefly, 7 known γ -ray pulsars have rather different γ -ray luminosity even their spin-down powers and ages are so similar (e.g. Geminga and PSR 1055-52). Their pulse shapes also differ so much. It is clear that there are other intrinsic parameters to control these observed properties (gamma-ray luminosity, pulse shape, spectrum etc).

In this paper, we re-study the gamma-ray emission from the outer gaps of the rotation-powered pulsars by using a new outer gap model. We follow the idea of self-sustained outer gap given by Zhang & Cheng (1997). However, we take the magnetosphere geometry as well as the average properties of the entire outer gap into consideration and show that the fractional size of the outer gap is a function of period, magnetic field and magnetic inclination angle. In fact, the effect of the inclination angle on the γ -ray emission have been considered in other versions of the outer gap models. For example, Romani & Yadigaroglu (1995) and Yadigaroglu & Romani (1995) took the magnetic inclination angle into account in their outer gap models, Hirotani and his colleagues also included the magnetic inclination angle in their calculation of the outer gap model (e.g. Hirotani, 2001; Hirotani & Shibata 2001; Hirotani & Shibata 2002;

Hirofani, Harding, Shibata 2003). Different from the treatment of Romani & Yadigaroglu (1995) who considered it in a less analytic way, we give an explicit expression for the fractional size of the outer gap. In section 2, we describe the revised outer gap model. We estimate the γ -ray luminosity for rotation-powered pulsars and compare them with the observed data in section 3. In section 4, we derive the death lines of the pulsars with outer gaps. Finally, we give briefly our conclusion and discussion.

2. The Outer Gap Model

2.1. Magnetospheric Geometry

For an oblique magnetic dipole rotator with an angular velocity Ω and the magnetic moment vector μ , let its spin axis be along the Oz axis, μ be in the plane xOz and α be the angle between Ω and μ . In polar coordinates,

$$\Omega = \Omega(\cos \theta \hat{r} - \sin \theta \hat{\theta}) . \quad (2)$$

and $\mu = \mu(\cos(\theta - \alpha)\hat{r} - \sin(\theta - \alpha)\hat{\theta})$. Corresponding magnetic field is

$$\mathbf{B}(\mathbf{r}) = \frac{\mu}{2r^3}(\mathbf{2} \cos(\theta - \alpha)\hat{r} + \sin(\theta - \alpha)\hat{\theta}) , \quad (3)$$

where $\mu = B_p R^3/2$, B_p and R are the stellar radius and surface magnetic field at pole (see, for example, Zhang & Harding 2000). \hat{r} and $\hat{\theta}$ are the unit vectors of radial and polar angle directions respectively. For a pulsar with a period P and a period derivative \dot{P} , the B_p is estimated by

$$B_p \approx 6.4 \times 10^{19} (P\dot{P})^{1/2} \text{ G} . \quad (4)$$

It is believed that the outer gap is extended from its inner boundary to the light cylinder (CHR I). For the oblique magnetic dipole rotator, the polar angle, θ_c , in which the last open field line is tangent to the light cylinder given by (Kapoor & Shukre 1998)

$$\tan \theta_c = -\frac{3}{4 \tan \alpha} (1 + (1 + 8 \tan^2 \alpha/9)^{1/2}) , \quad (5)$$

corresponding radius is

$$r_c = \frac{R_L}{\sin \theta_c} , \quad (6)$$

it should be noted that $\theta_c = \pi/2$ and $r_c = R_L$ for an aligned magnetic dipole. Along the last open field line, the relation

$$\frac{\sin^2(\theta - \alpha)}{r} = \frac{\sin^2(\theta_c - \alpha)}{r_c} \quad (7)$$

is valid. The inner boundary of the outer gap is estimated by the null charge surface, which is defined by $\Omega \cdot \mathbf{B} = 0$. In the two dimensional case, the null charge surface can be described by (r_{in}, θ_{in}) . By definition, we have

$$\tan \theta_{in} = \frac{1}{2} (3 \tan \alpha + \sqrt{9 \tan^2 \alpha + 8}) , \quad (8)$$

while r_{in} is estimated along the last open field line, which gives using Eq. (7)

$$\frac{r_{in}}{R_L} = \frac{\sin^2(\theta_{in} - \alpha)}{\sin \theta_c \sin^2(\theta_c - \alpha)} . \quad (9)$$

Therefore, the outer gap extended from r_{in} to r_c along the last open field line for the oblique magnetic dipole. In such a geometry, the Goldreich-Julian current is roughly

$$\dot{N}_{GJ} \approx \frac{\Omega^2 R^3 B_p}{2ec} a(\alpha) \cos \alpha , \quad (10)$$

where $a(\alpha) = \sin \theta_c \sin^2(\theta_c - \alpha)$.

2.2. X-ray Field in the Magnetosphere

The observed spectra of X-ray emission from some rotation-powered pulsars indicate that there are at least two kinds of X-ray spectra. One consists of soft X-rays, which can be fitted by a black-body spectrum with a single temperature, combined with a hard X-ray spectrum with a power-law distribution. Another has only a thermal spectrum. It is believed that the non-thermal components are most likely of magnetospheric origin, while the origin of thermal components is less clear because there are several mechanisms for production of thermal emission in the soft X-ray bands. Observationally, the bulk of the soft X-ray emission between ~ 0.1 - 1 keV is well fitted by a double black-body at two different temperatures. The ratio of the area of the hotter component to the colder one is typically very small (\sim a few $\times 10^{-5}$ to a few $\times 10^{-5}$). The colder component has been explained as resulting from thermal cooling, while

the hotter one most likely comes from bombardment by high-energy particles (Greiveldinger et al. 1996). It is expected that pulsars younger than $\sim 10^5$ yr have significant neutron star cooling components. We describe briefly the main possible mechanisms of X-ray production as below.

2.2.1. Thermal X-ray emission from neutron star cooling

For thermal X-ray emission due to neutron star cooling, it is believed that its spectrum is expressed with a modified black-body, we approximate it as a black-body spectrum with a temperature T_c . Because the neutron star cooling concerns many different mechanisms, the estimate of the temperature T_c exists uncertainties for different models. For example, the temperature can be expressed as (Romani 1996)

$$T_{c,6} \approx \left(\frac{\tau}{10^5 \text{yr}} \right)^{-0.05} \exp(-\tau/10^6 \text{yr}) \text{ K} , \quad (11)$$

where $T_{c,6}$ is the temperature of the neutron star cooling in units of 10^6 K and τ is the neutron star's age. Above expression is valid for τ less than several 10^6 yr. Zhang & Harding (2000) used a expression to approximate the temperature T_c as

$$T_{c,6} = \begin{cases} 10^{-0.23} \tau_6^{-0.1} & \tau \leq 10^{5.2} \text{yr} \\ 10^{-0.55} \tau_6^{-0.5} & \tau > 10^{5.2} \text{yr} , \end{cases} \quad (12)$$

where τ_6 is the age in units of 10^6 yr. Based on the cooling model derived from Tsuruta (1998) (in this model, the effects of the magnetic field on thermal conductivity are included, but polar cap heating is not considered), Hibschan & Arons (2001) use a temperature model as follow

$$T_{c,6}(t) = \begin{cases} 10^{0.05} \tau_6^{-0.1} & \tau < 10^7 \text{yr} \\ 10^{0.325} \tau_6^{-0.375} & \tau > 10^7 \text{yr} , \end{cases} \quad (13)$$

where t is the spin-down age of the neutron star. For the hotter component originating from the bombardment by high-energy particles, the high-energy particles are produced either in the polar cap region or in outer gap region, resulting in different properties of the hotter component. In other words, the origin of the relativistic particles, which bombard the stellar surface to produce thermal hotter X-rays, depends on the models.

2.2.2. X-ray emission from polar cap heating

In the polar cap models, the return relativistic particles are produced in the polar gap (e.g. Ruderman & Sutherland 1975; Arons 1981). Recently, Zhang & Harding (2000) considered full polar cap cascade scenario of the polar cap model and estimated thermal

X-ray luminosity using a self-consistent polar cap heating in the Harding & Muslimov (1998) model (a relevant recent study see Hibschan & Arons 2001). Further, Harding & Muslimov (2001) studied the effect of pulsar polar cap heating produced by

positron returning from upper pair formation front, the polar cap heating produces a thermal X-ray emission with a temperature, $T_{pc} = (L_{pc}/\sigma A)^{1/4}$,

$$T_{pc,6} = \begin{cases} 2.46 \left(\frac{P_{0.1}}{\tau_6} \right)^{\frac{1}{28}} \left(\frac{\cos^2 \alpha}{a(\alpha)} \right)^{\frac{1}{4}} & \text{for } P_{0.1}^{\frac{9}{4}} < 0.5 B_{12} \\ 2.51 P_{0.1}^{\frac{1}{8}} \left(\frac{\cos^2 \alpha}{a(\alpha)} \right)^{\frac{1}{4}} & \text{for } P_{0.1}^{\frac{9}{4}} > 0.5 B_{12} , \end{cases} \quad (14)$$

where L_{pc} is the X-ray luminosity emitted from the polar caps, σ is the Stefan constant, and A is the heated area of the returning positrons from the polar gaps. For the canonical polar cap area, $A = A_{pc} = \pi R^2 (\Omega R/c)$. In the above expressions, $P_{0.1} = P/0.1$ s, $B_{12} = B/10^{12}$ G and $\tau_6 = 10^6 \tau$ yr is the pulsar age, we have used the analytic estimate of X-ray luminosity given by Harding and Muslimov (2001) and A_{pc} , so the above expressions are valid for normal pulsars with $\tau \leq 10^7$ yr.

2.2.3. X-ray emission from outer gap heating

In the outer gap models, part of the relativistic particles from the outer gap will collide the stellar surface, producing the thermal X-rays. These relativistic inflowing particles from the outer gap radiate away much of their energy before reaching the polar cap (Zhang & Cheng 1997, Zhu et al. 1997; Wang et al. 1998; Cheng & Zhang 1999). The residual energy of the charged particles striking the polar cap is

$$E_e(R) \approx \left(\frac{2e^2 c}{mc^3 R_L} \ln \frac{r}{R} \right)^{-1/3} . \quad (15)$$

These particles collide with the polar cap at a rate of $\dot{N}_e = f \dot{N}_{GJ}$, where \dot{N}_{GJ} is the Goldreich-Julian

particle flux (Goldreich & Julian 1969) and estimated by Eq. (10). Therefore the polar cap is heated and radiates X-rays with a luminosity ($L_X \approx f E_e(R) \dot{N}_{GJ}$)

$$L_X \approx 2.3 \times 10^{31} f B_{12} P^{-5/3} R_6^3 \left(\ln \frac{r}{R} \right)^{-1/3} a(\alpha) \cos \alpha \text{ ergs s}^{-1} \quad (16)$$

These X-rays have a typical temperature $T_h = (L_X / A \sigma_{SB})^{1/4}$ with

$$T_h \approx 5.0 \times 10^6 P^{-1/6} B_{12}^{1/4} \left(\ln \frac{r}{R} \right)^{-1/12} (\sin^2(a(\alpha) \cos \alpha))^{1/4} \text{ K} \quad (17)$$

where $A \sim \pi f \Omega R^3 / c$ is the area of the stellar surface bombarded by the return current and σ_{SB} is Stefan's constant. Generally, a part of these X-rays will escape along the open magnetic field lines. Following Cheng & Zhang (1999), we use L_X^h as the luminosity of the X-rays escaping along the open field lines and introduce a parameter $\xi = L_X^h / L_X$ (the estimate of ξ see Cheng & Zhang (1999)). Most of X-rays are reflected back to the stellar surface because of the cyclotron resonant X-ray reflecting mirror. This process transfers emitted polar cap X-ray energy to the entire surface of the neutron star. Finally, the X-rays from the entire surface of the neutron star are emitted with a temperature of $T = (L_X^s / 2\pi R^2 \sigma)^{1/4}$, where $L_X^s = (1 - \xi) L_X$. Corresponding characteristic temperature is $T_s = (L_X^s / 4\pi R^2 \sigma_{SB})^{1/4}$, which gives

$$T_s \approx 4.2 \times 10^5 (1 - \xi)^{1/4} f^{1/4} P^{-5/12} B_{12}^{1/4} \left(\ln \frac{r}{R} \right)^{-1/12} (a(\alpha) \cos \alpha)^{1/4} R_6^{1/4} \text{ K} \quad (18)$$

2.2.4. Average energy of X-rays

We consider two possible cases for the thermal X-rays from the stellar surface. In the first case, the thermal X-ray are produced by both the neutron star standard cooling mechanism and polar cap heating, the temperatures are given by Eq. (12) or Eq. (13) for the standard cooling mechanism and Eq. (14) for the polar cap heating. In the second case, the thermal X-ray come from the bombardment of the relativistic particles from the outer gap (e.g. Zhang & Cheng 1997), the corresponding temperatures are given by Eqs. (18)

and (17). Assuming these X-ray can be approximated as the black-body, their spectrum can be expressed as

$$F_X(E_X) = C \left[\frac{E_X^2}{e^{E_X/kT_1} - 1} + \frac{A}{4\pi R^2} \frac{E_X^2}{e^{E_X/kT_2} - 1} \right] \quad (19)$$

where R is the stellar radius, A is the area of polar cap being heated, and T_1 and T_2 are the temperatures being whole stellar surface and polar cap respectively. Using above expression, we can estimate the average X-ray energy as

$$\langle E_X \rangle = \frac{\int F_X(E_X) E_X dE_X}{\int F_X(E_X) dE_X} = \frac{\pi^4}{30\zeta(3)} \frac{1 + (A/4\pi R^2)(T_2/T_1)^4}{1 + (A/4\pi R^2)(T_2/T_1)^3} (kT_1) \quad (20)$$

where $\zeta(x)$ is the Zeta function and $\zeta(3) \approx 1.2$. For the different mechanisms of thermal X-rays from the stellar surface, the dependence of $\langle E_X \rangle$ on the basic parameters of pulsars is different.

We consider two possible cases for estimating the average X-ray energy: (i) thermal X-rays are produced by the standard neutron star cooling and polar cap heating, we have

$$\langle E_X^{pc} \rangle \approx 2.7 k T_c \frac{1 + (R/4R_L)(T_{pc}/T_c)^4}{1 + (R/4R_L)(T_{pc}/T_c)^3} \quad (21)$$

and (ii) the thermal X-rays are produced by the bombardment of the relativistic particles from the outer gap, we have

$$\langle E_X^{og} \rangle \approx 2.7 k T_s \frac{2 - \xi}{1 - \xi} \quad (22)$$

2.3. The Fractional Size of an Outer Gap

In two dimensional geometry, the fractional size of the outer gap is an important parameter for the γ -ray production in the outer gap. According to Zhang & Cheng (1997), the parallel electric field in the outer gap can be approximated as

$$E_{||} = f^2 B(r) \left(\frac{s}{R_L} \right) \quad (23)$$

where f is the fractional size of the outer gap, $B(r) = B_p (1 + 3 \cos^2(\theta - \alpha))^{1/2} R^3 / r^3$ is the magnetic field strength at the radius, r , to the star,

R_L is the radius of the light cylinder and s is the curvature radius which is (Lesch et al. 1998)

$$s(\theta, \theta_s) = \frac{R}{3} \frac{\sin(\theta - \alpha)}{\sin^2(\theta_s - \alpha)} \frac{(1 + 3 \cos^2(\theta - \alpha))^{\frac{3}{2}}}{1 + \cos^2(\theta - \alpha)}, \quad (24)$$

where θ_s is the polar angle at the stellar surface. Equation (25) can be written by

$$s(\theta, \theta_s) = \sqrt{r R_L} W(\alpha, r) \quad (25)$$

with

$$W(\alpha, r) = \frac{4}{3} \frac{[1 - \frac{3}{4}a(\alpha)\frac{r}{R}]^{3/2}}{\sqrt{a(\alpha)}(1 - \frac{1}{2}a(\alpha)\frac{r}{R_L})} \quad (26)$$

where $a(\alpha) = \sin^2(\theta_c - \alpha) \sin \theta_c$. This electric field will accelerate the electrons/positrons to relativistic energy in the outer gap. Because these accelerated particles will lose their energy through curvature radiation, their Lorentz factor is estimated by using $eE_{||}c = (2/3)e^2c\gamma^4/s^2$, which gives

$$\gamma(r) \approx 2.84 \times 10^7 f^{\frac{1}{2}} B_{12}^{\frac{1}{4}} P^{-\frac{1}{4}} R_6^{\frac{3}{4}} \left(\frac{r}{R_L}\right)^{-\frac{3}{8}} \left(\frac{\sqrt{1 + 3 \cos^2(\theta - \alpha)}}{2}\right)^{\frac{1}{4}}, \quad (27)$$

where P is the pulsar period in units of second and R is the stellar radius in units of 10^6 cm. The characteristic energy of the γ -ray photons in the outer gap can be approximated as

$$E_\gamma \approx 143 f^{\frac{3}{2}} B_{12}^{\frac{3}{4}} P^{-\frac{7}{4}} \left(\frac{\sqrt{1 + 3 \cos^2(\theta - \alpha)}}{2}\right)^{\frac{3}{4}} \left(\frac{r}{R_L}\right)^{-\frac{3}{2}} \left(\frac{s}{R_L}\right)^{-\frac{1}{4}} R_6^{\frac{9}{4}} \text{ MeV}, \quad (28)$$

Inside the outer gap, the curvature photons interact with the thermal X-rays from the stellar surface to produce e^\pm pairs through photon-photon pair production process, sustaining the outer gap. This pair production condition is

$$\langle E_X \rangle E_\gamma (1 - \cos(\theta_{X\gamma})) = 2(m_e c^2)^2, \quad (29)$$

where $\langle E_X \rangle$ is the average X-ray energy which is estimated below, $\theta_{X\gamma}$ is the angle between the emission directions of curvature photons and the

thermal X-rays. We assume that the curvature photons are emitted along the negative direction of the magnetic field and the thermal X-rays along the radial direction, then we have

$$\cos \theta_{X\gamma} = -\frac{2 \cos(\theta - \alpha)}{(3 \cos^2(\theta - \alpha) + 1)^{1/2}}, \quad (30)$$

where θ is the polar angle at radius r . Putting Eq. (7) into Eq.(30), we have

$$\cos \theta_{X\gamma} = -\left(\frac{1 - (r/R_L)a(\alpha)}{(1 - (3/4)(r/R_L)a(\alpha))}\right)^{1/2}, \quad (31)$$

where $a(\alpha) = \sin^2(\theta_c - \alpha) \sin \theta_c$.

We consider the fractional sizes of the outer gap corresponding to two possible average energies of X-rays. In the first case, X-rays are produced by the neutron star cooling and polar cap heating. The average X-ray energy is given by Eq. (21), and the fractional size of the outer gap is

$$f(r, \alpha) \approx 6.9 B_{12}^{-\frac{1}{2}} P^{\frac{7}{8}} < E_X >_{0.1}^{-\frac{2}{3}} G_{pc}(r, \alpha) \quad (32)$$

with

$$G_{pc} = W^{1/6} \left(\frac{2}{1 - \cos \theta_{x\gamma}}\right)^{2/3} \left(\frac{r}{R_L}\right)^{\frac{13}{12}} \left(\frac{\sqrt{1 + 3 \cos^2(\theta - \alpha)}}{2}\right)^{-\frac{1}{2}}, \quad (33)$$

where $\langle E_X \rangle_{0.1} = \langle E_X \rangle / 0.1$ keV. Because the temperature of the polar cap heating is greater than that of the neutron star cooling, equation (21) can be approximated as $\langle E_X \rangle \approx 2.7 k T_c$. Using Eqs. (12) and (13) respectively, we have

$$\langle E_X \rangle_{0.1} \approx \begin{cases} 32.9 \left(\frac{P}{P}\right)^{-0.1} & \text{for } \dot{P}_{-15} \geq 10^2 P \\ 5.2 \times 10^6 \left(\frac{P}{P}\right)^{-0.5} & \text{for } \dot{P}_{-15} < 10^2 P \end{cases} \quad (34)$$

and

$$\langle E_X \rangle_{0.1} \approx \begin{cases} 7.49 \left(\frac{P}{P}\right)^{-0.11} & \text{for } \dot{P}_{-15} \geq 1.6 P \\ 7.33 \times 10^4 \left(\frac{P}{P}\right)^{-\frac{15}{40}} & \text{for } \dot{P}_{-15} < 1.6 P \end{cases}, \quad (35)$$

where $\dot{P} = 10^{-15} \dot{P}_{-15}$ is the period derivative of a pulsar in units of s s^{-1} . In the second case, the thermal X-ray come from the bombardment of the

relativistic particles from the outer gap. Putting Eqs. (28) and (20) into Eq. (29), we have

$$f(r, \alpha) \approx 5.2 B_{12}^{-4/7} P^{26/21} R_6^{10/7} G(r, \alpha) \quad (36)$$

with

$$G(r, \alpha) = \left[\frac{2}{1 - \cos \theta_{X\gamma}} \right]^{\frac{4}{7}} \left(\ln \frac{r}{R} \right)^{\frac{1}{21}} \left(\frac{r}{R_L} \right)^{\frac{13}{14}} \left(\frac{W}{a(\alpha) \cos \alpha} \right)^{\frac{1}{7}} \left(\frac{\sqrt{1 + 3 \cos^2(\theta - \alpha)}}{2} \right)^{-\frac{3}{7}} \quad (37)$$

Obviously f is a function of r as well as the inclination angle α in the two cases.

It is believed that an outer gap start at the null charge surface ($\mathbf{\Omega} \cdot \mathbf{B} = 0$), which defines the inner boundary of the outer gap and the radial distance is r_{in} . From Eq. (32) or Eq. (36), the fractional size reaches a minimum at the radius (r_{in}) of the inner boundary, and then increases with radius for a given pulsar. Therefore, the fractional size of the outer gap at the radius r_{in} determine whether or not the outer gap exists. If $f(r_{in}, \alpha) > 1$, it means that the pulsar does not exist any outer gap. In other words, a pulsar with $f(r_{in}, \alpha) \leq 1$ should have its outer gap and will emit high-energy photons produced in the outer gap. As the radius increases, the fractional size of the outer gap increases. For a pulsar with $f(r_c, \alpha) \leq 1$, the outer gap will extend from r_{in} to r_c . However, the pulsar with $f(r_c, \alpha) > 1$ will stop at some radius r_b , in which $f(r_b, \alpha) = 1$. In order to explain the average properties of high-energy photon emission from the outer gap, we assume that high-energy emission at a average radius $\langle r \rangle$ represents the typical emission of high-energy photons from a pulsar. The average radius is given by

$$\langle r \rangle = \frac{\int_{r_{in}}^{r_{max}} f(r, \alpha) r dr}{\int_{r_{in}}^{r_{max}} f(r, \alpha) dr}, \quad (38)$$

where $r_{max} = \min(r_c, r_b)$. The average gap size is approximated as $f(\langle r \rangle, P, B)$ by substituting $\langle r \rangle$ into Eq. (32) and Eq. (36) and in general a function of P , B and α because r_c is a function of P and α , and r_b are function of P , B and α .

Generally, the inclination angles for the pulsars are not known well, so we consider the changes of the fractional size with the inclination angle. Let us define the following function:

$$f(\langle r \rangle, P, B) = \eta(\alpha, P, B) f_o(P, B) \quad (39)$$

where $f_o(P, B) = 5.5 P^{26/21} B_{12}^{-4/7}$ is the fractional size of outer gap by ignoring the effect of inclination angle (Zhang & Cheng 1997). So the effect of inclination angle should exhibit in the function $\eta(\alpha, P, B)$. In Fig. 1, we consider the variation of outer gap size with magnetic inclination angle for various P and B . In panel A of Fig. 1, the upper (lower) solid line is the self-consistent outer gap model (the cooling X-ray outer gap model) with $P = 0.1s$ and the upper (lower) dashed line is the self-consistent outer gap model (the cooling X-ray outer gap model) with $P = 0.3s$ respectively. The magnetic field is chosen to be $3 \times 10^{12}G$. In panel B of Fig. 1, the upper (lower) solid line is the self-consistent outer gap model (the cooling X-ray outer gap model) with $B_{12} = 3.0$ and the upper (lower) dashed line is the self-consistent outer gap model (the cooling X-ray outer gap model) with $B_{12} = 1.5$ respectively. The period is chosen to be $0.2s$. We can see that η varies about a factor of 2 for the self-consistent model but becomes rather constant for the cooling X-ray outer gap model.

3. High-Energy Gamma-ray Luminosity

EGRET has observed six pulsars with high confidence and three possible radio pulsars to emit high-energy gamma-rays above 100 MeV (see Thompson 2001 for a review). One of these possible radio pulsars is a millisecond pulsar PSR J0218+4232 (Kuiper et al. 2000). Therefore, there could be eight young pulsars to emit high-energy gamma-rays observed by EGRET. It should be noted that PSR B1509-58 is also a gamma-ray pulsar. But it is seen only up to 10 MeV by COMPTEL (Kuiper et al. 1999) and not above 100 MeV by EGRET. The observed gamma-ray luminosity of a pulsar is $L_{\gamma}^{obs} = 4\pi d^2 \zeta F_{\gamma}$, where F_{γ} is the observed gamma-ray flux, d is the distance to the pulsar and ζ is the gamma-ray beaming fraction ($0 < \zeta \leq 1$), which is defined as the ratio of the beaming solid angle to 4π . Two parameters are highly uncertain: the distance d and the beaming fraction ζ . It is commonly assumed that $\zeta = 1/4\pi$ in estimating the observed gamma-ray luminosity. However, the γ -ray beaming fraction should be different for various γ -ray pulsars, which is a function of the magnetic inclination angle as well as the size of the outer gap. Some approximation forms of beaming fraction have been given (Yadigaroglu & Romani 1995; Ro-

mani 1996; Zhang, Zhang & Cheng 2000). How accurate of these approximation forms are not known. Furthermore, we want to emphasize that in addition the beaming fraction the distance estimate also affects the observed gamma-ray luminosity strongly. For example, the recent measurement shows that the distance to the Vela pulsar is 294_{-50}^{+76} pc (Cavareo et al. 2001), which is less than previous value (500 pc) derived from radio observation. Hence the gamma-ray luminosity of the Vela could be a factor of 2 lower than that given in Thompson et al.(2001). In this paper, we just want to see how the inclination affects the gamma-ray luminosity. For simplification, we use a common assumption of $\zeta = 1/4\pi$ in order to compare with the observed data given by Thompson et al.(2001).

Because high-energy gamma-rays are mainly produced from the outer gap in our model, we will compare our expected gamma-ray luminosities with those of the gamma-ray pulsars which emit gamma-rays observed by EGRET. In our model, gamma-ray luminosity for each gamma-ray pulsar depends on period, magnetic field and the magnetic inclination. However, the magnetic inclination angles are not known well. Once the average fractional size of the outer gap for a pulsar is estimated, the gamma-ray luminosity can be approximated as

$$L_\gamma \approx f^3(\langle r \rangle) L_{sd} \quad , \quad (40)$$

where $L_{sd} = 4\pi^2 I \dot{P} / P^3$ is the spin-down luminosity of the pulsar and $I = 10^{45}$ g cm². Using Eq. (4) and putting Eq. (36) into Eq. (40), we have

$$L_\gamma = L_{\gamma,0} \eta^3(\alpha, P, B) \quad (41)$$

with

$$L_{\gamma,0} \approx 1.36 \times 10^{33} B_{12}^{2/7} P^{-2/7} \quad . \quad (42)$$

It is obviously that Eq. (42) is the same as that given by Zhang & Cheng (1997).

In principle, we can estimate γ -ray luminosity for a pulsar with a known inclination angle. However, it is difficult to estimate γ -ray luminosity for all canonical pulsars because only the inclination angles of a few of pulsars are known. Therefore, we find out statistically the relation between γ -ray luminosity and pulsar spin-down power for the canonical pulsars using Monte Carlo method.

The details of this Monte Carlo method is given by Cheng & Zhang (1998) and Zhang, Zhang & Cheng (2000). We use the following assumptions for generating the Galactic pulsar population:

- (i) The pulsars are born at a rate ($\dot{N}_{NS} \sim (1 - 2)$ per century) with spin periods of $P_0 = 10$ ms.
- (ii) The initial position for each pulsar is estimated from the distributions $\rho_z(z) = (1/z_{\text{exp}}) \exp(-|z|/z_{\text{exp}})$ and $\rho_R(R) = (a_R/R_{\text{exp}}^2) R \exp(-R/R_{\text{exp}})$, where z is the distance from the Galactic plane, R is the distance from the Galactic center, $z_{\text{exp}} = 75$ pc, $a_R = [1 - e^{-R_{\text{max}}/R_{\text{exp}}}(1 + R_{\text{max}}/R_{\text{exp}})]^{-1}$, $R_{\text{exp}} = 4.5$ kpc and $R_{\text{max}} = 20$ kpc (Paczynski 1990; Sturmer & Dermer 1996).
- (iii) The initial magnetic fields are distributed as a Gaussian in $\log B$ with a mean $\log B = 12.52$ and a dispersion $\sigma_B = 0.35$. We ignore any field decay for these rotation-powered pulsars.
- (iv) The initial velocity of each pulsar is the vector sum of the circular rotation velocity at the birth location and a random velocity from the supernova explosion (Paczynski 1990; Cheng & Zhang 1998), the circular velocity is determined by Galactic gravitational potential and the random velocity are distributed as a Maxwellian distribution with dispersion of three dimensional velocity $\sigma_V = \sqrt{3} \times 100$ km/s (Lorimer, Bailes & Harrison 1997).
- (v) A random distribution of magnetic inclination angles is used (e.g. Biggs 1990). However, the values of α subject to two constraints. First the photon energy given in Eq. (28) cannot higher than the total potential drop of the outer gap. Second r_{in} in Eq. (9) must be larger than the stellar radius.

The pulsar period at time t can be estimated by

$$P(t) = \left[P_0^2 + \left(\frac{16\pi^2 R_{NS}^6 B^2}{3Ic^3} \right) t \right]^{1/2} \quad , \quad (43)$$

where, R_{NS} is the neutron star radius and I is the neutron star moment of inertia. The period derivative (\dot{P}) can be determined by

$$P\dot{P} = (8\pi^2 R_{NS}^6 / 3Ic^3) B^2 . \quad (44)$$

Furthermore, the pulsar position at time t is determined following its motion in the Galactic gravitational potential. Using the equations given by Paczynski (1990) for given initial velocity, the orbit integrations are performed by using the 4th order Runge Kutta method with variable time step (Press et al. 1992) on the variables R , V_R , z , V_z and ϕ . Then the sky position and the distance of the simulated pulsar can be calculated.

We now consider the observational selection effects. First, we consider radio selection effects in order to generate a pulsar population detectable at the radio band: the pulsar must satisfy that its radio flux is greater than the radio survey flux threshold and its broadened pulse width is less than the rotation period (e.g. Sturmer & Dermer 1996). The pulsar which satisfies

$$L_{400}/d^2 \geq S_{\min} \quad (45)$$

is considered to be a radio-detectable pulsar, where L_{400} is the radio luminosity at 400 MHz and d is the distance to the pulsar. The radio beaming fraction can be expressed as (Emmering & Chevalier 1989)

$$f_r(\omega) = (1 - \cos \omega) + (\pi/2 - \omega) \sin \omega , \quad (46)$$

where $\omega = 6^\circ.2 \times P^{-1/2}$ (e.g. Biggs, 1990) is the half-angle of the radio emission cone. Then, following Emmering & Chevalier (1989), a sample pulsar with a given period P is chosen in one out of $f_r(P)^{-1}$ cases using the Monte Carlo method. Second, we consider the γ -ray selection effects. According to Zhang, Zhang and Cheng (2000), we use

$$S_\gamma(> 100\text{MeV}) \geq 1.2 \times 10^{-10} \text{ erg cm}^{-2}\text{s}^{-1} \quad (47)$$

as the minimum detectable γ -ray energy flux, which corresponds roughly to the faintest sources with $(TS)^{1/2} > 5$.

Using above method, we can generate a γ -ray pulsar population. In Fig. 2, we show the relation between γ -ray luminosity and the spin-down power for the simulated γ -ray pulsar population by

using this new outer gap model, where the birth rate of the neutron stars is assumed to be $1/200$ yr and the pulsar ages are limited to be not greater than 10^7 yr. In this figure, we show two pulsar populations, one is the population which radio selection effects of the pulsar with outer gaps are taken into account (i.e. radio-loud γ -ray pulsars see shaded circles in Fig.2), and the other population is the radio-quiet γ -ray pulsars (see open circles in Fig.2).

There are several interesting features in this figure. First there is a rather sharp boundary on the left of the population. In fact this boundary is given by

$$L_\gamma = L_{sd}. \quad (48)$$

This relation/boundary results from the fact that γ -ray pulsars terminate at $f(< r) = 1$ and $r_{\max} = r_b$ in Eq. (38). Similarly the second rough boundary appearing at the bottom of the population is caused by $f(< r) = 1$ and $r_{\max} = r_c = r_L$. Zhang & Cheng (1997) has estimated the fractional gap size by assuming that the typical distance from the gap to the star is $\sim r_L$, they obtained $f = 5.5 B_{12}^{-4/7} P^{26/21}$. When we substitute this relation into L_{sd} , we obtain

$$L_{sd}^{crit} = 1.5 \times 10^{34} P^{1/3} \text{ ergs}^{-1}. \quad (49)$$

It is important to note that $L_\gamma = L_{sd}$ as $L_{sd} \lesssim L_{sd}^{crit}$.

In such a pulsar population, the best fit gives following relation

$$\log L_\gamma \approx 20.42 + 0.38 \log L_{sd} \quad (50)$$

for the pulsars with outer gaps. When taking the radio selection effects, the slope between L_γ and L_{sd} becomes flatter, which is $\log L_\gamma \approx 23.25 + 0.30 \log L_{sd}$. We will consider the statistics of the pulsars with outer gaps in the rest of this section. In order to show the important effect of the magnetic inclination angle on γ -ray luminosity, we show the change of $\log_{10}(L_\gamma/L_{\gamma,0})$ with $\log_{10} L_\gamma$ in Fig. 3. The best fit indicates that $\log_{10}(L_\gamma/L_{\gamma,0}) \propto 0.9 \log_{10} L_\gamma$. For comparison, we show the result given by Zhang & Cheng (1997), which is independent with the inclination angle.

It is very interesting that the slope between L_γ and L_{sd} will become steeper when the minimum

detectable γ -ray energy flux decreases. For example, we obtain a γ -ray pulsar population using GLAST threshold ($1.8 \times 10^{-12} \text{ erg cm}^{-2} \text{ s}^{-1}$), the best fit gives

$$\log L_\gamma \approx 17.45 + 0.46 \log L_{sd} \quad . \quad (51)$$

That is $L_\gamma \propto L_{sd}^{1/2}$. In this pulsar population, we have $\log_{10}(L_\gamma/L_{\gamma,0}) \propto 0.92 \log L_\gamma$.

For comparison, we show our results given by Eqs. (50) and (51) and the observed high-energy photon luminosities of eight γ -ray pulsars in Fig. 4. In this figure, the observed data are taken from Thompson (2001), which derived from detected fluxes above 1 eV assuming the solid angle of photon beaming of 1 sr. From Fig. 5, it can be seen that both our results given by Eqs. (50) and (51) are consistent with the observed data. For the result fitting the simulated γ -ray pulsar population with EGRET threshold, the expected slope (0.38) is flatter than the observed one (0.46).

Finally we would like to point out that this new outer gap model predicts many more weak γ -ray pulsars whose γ -ray luminosities can be as low as $10^{32} \text{ erg s}^{-1}$ (cf. Fig. 2) than previous outer gap models (Zhang & Cheng 1997; Cheng & Zhang 1998). The main reason for existing these weak γ -ray pulsars is that instead of using the outer gap size at half of light cylinder radius to determine if the outer gap can exist, the new model allows the outer gap to survive when the fractional size of outer gap at the null charge surface for a given pulsar is less than unity. This prediction allows the age of γ -ray pulsars to extend to a few million years old. In fact, more detail Monte Carlo simulation results show that most of these weak γ -ray pulsars have ages between 0.3-3 million years old and many of them are located at higher galactic latitude. Most of these weak γ -ray pulsars their radio beam could miss the Earth and they could contribute to the unidentified γ -ray sources in high galactic latitude (Cheng et al. 2003). These predictions can be verified by GLAST.

4. The Death lines of Pulsars with Outer Gaps

We now consider the condition which the outer gap of a pulsar exists. For an outer gap, the inner boundary is estimated as the null charge surface where the magnetic field lines are perpendicular to

the rotation axis. From Eq. (36) or (32), $f(r, \alpha)$ reaches minimum at $r = r_{in}$. In other words, if the fractional size of the outer gap at r_{in} is larger than unity, then the outer gap would not exist. Therefore, we can estimate the death lines of the pulsars with outer gaps by using $f(r_{in}, \alpha) = 1$. It should be noted that $G_{pc}(r_{in}, \alpha)$ in Eq. (32) or $G(r_{in}, \alpha)$ in Eq. (36) is only the function of α because r_{in}/R_L only depends on α (see Eq. (9)). For the case of X-rays produced by the neutron star cooling and polar cap heating, we obtain from $f(r_{in}, \alpha) = 1$

$$\log \dot{P} = \begin{cases} 3.1 \log P - A_1(\alpha) & \text{for } \log P \leq 13 + \log \dot{P} \\ \frac{15}{7} \log P - A_2(\alpha) & \text{for } \log P > 13 + \log \dot{P} \end{cases} \quad (52)$$

for the temperature given by Zhang & Harding (2000), where $A_1(\alpha) = 12.87 - 3.16 \log G_{pc}(\alpha)$ and $A_2(\alpha) = 12.92 - \frac{12}{7} \log G_{pc}(\alpha)$; and

$$\log \dot{P} = \begin{cases} 3.1 \log P - B_1(\alpha) & \text{for } \log P \leq 14.8 + \log \dot{P} \\ \frac{7}{3} \log P - B_2(\alpha) & \text{for } \log P > 14.8 + \log \dot{P} \end{cases} \quad (53)$$

for the temperature used by Hibschan & Arons (2001), where $B_1(\alpha) = 13.46 - 3.16 \log G_{pc}(\alpha)$ and $B_2(\alpha) = 13.95 - 2 \log G_{pc}(\alpha)$. For the case of X-rays produced by the bombardment of the returning particles from the outer gap, we have

$$\log \dot{P} = \frac{10}{3} \log P - 13.02 + \frac{7}{2} \log G(\alpha) \quad . \quad (54)$$

Obviously, the death lines depend on the magnetic inclination angle.

Although the magnetic inclination angle of each pulsar has not been determined well, the distribution of the magnetic inclination angle can be estimated from the statistical polarization study of the radio pulsars. It was generally believed that the parent distribution of the magnetic inclinations satisfy a uniform distribution (Gunn & Ostriker 1970; Gil & Han 1996). However, recent study by using polarization data of the radio pulsars indicate that the parent distribution of the magnetic inclinations satisfies a cosine-like distribution (Tauris & Manchester 1998). Therefore, we estimate the average value of $f(r_{in}, \alpha)$ in these two possible parent distributions of the magnetic inclinations, i.e.

$$\langle G(\alpha) \rangle = \int G(\alpha) U(\alpha) d\alpha / \int U(\alpha) d\alpha \quad , \quad (55)$$

where $U(\alpha)$ represents the distribution of the inclination angles. We consider uniform and cosine-like distributions of the inclination angles respectively. For the uniform distribution, we have $\langle G_{pc}(\alpha) \rangle \approx 0.32$ and $G(\alpha) = 0.38$. Therefore Eqs. (52), (53) and (54) become

$$\log \dot{P} = \begin{cases} 3.1 \log P - 14.43 & \text{for } \log P \leq 13 + \log \dot{P} \\ \frac{15}{7} \log P - 13.77 & \text{for } \log P > 13 + \log \dot{P} \end{cases}, \quad (56)$$

$$\log \dot{P} = \begin{cases} 3.1 \log P - 15.02 & \text{for } \log P \leq 14.8 + \log \dot{P} \\ \frac{7}{3} \log P - 14.94 & \text{for } \log P > 14.8 + \log \dot{P} \end{cases} \quad (57)$$

and

$$\log \dot{P} = \frac{10}{3} \log P - 14.60. \quad (58)$$

For cosine distribution, $\langle G_{pc}(\alpha) \rangle \approx 0.43$ and $G(\alpha) \approx 0.49$, Eqs. (52), (53) and (54) become

$$\log \dot{P} = \begin{cases} 3.1 \log P - 14.03 & \text{for } \log P \leq 13 + \log \dot{P} \\ \frac{15}{7} \log P - 13.55 & \text{for } \log P > 13 + \log \dot{P} \end{cases}, \quad (59)$$

$$\log \dot{P} = \begin{cases} 3.1 \log P - 14.62 & \text{for } \log P \leq 14.8 + \log \dot{P} \\ \frac{7}{3} \log P - 14.68 & \text{for } \log P > 14.8 + \log \dot{P} \end{cases} \quad (60)$$

and

$$\log \dot{P} = \frac{10}{3} \log P - 14.20. \quad (61)$$

Based on the original outer gap model (CHR I; CHR II), Chen & Ruderman (1993) have considered the death lines of Crab-like and Vela-like pulsars respectively. For the Crab-like pulsars, the death line is (see Eq. (25) of their paper)

$$\log \dot{P} = 4 \log P - 7. \quad (62)$$

For Vela-like pulsars, they introduced a parameter ξ to describe the expected variation of the magnetic field within the outer gap and assumed $\xi = 1/2$. In this case, the death line is (see Eq. (27) of their paper)

$$\log \dot{P} = 3.8 \log P - 10.2. \quad (63)$$

According to Zhang & Cheng (1997), we have from $f_s \approx 2.83 \times 10^{-4} P^{20/21} \dot{P}^{-2/7} = 1$

$$\log \dot{P} = \frac{10}{3} \log P - 12.42. \quad (64)$$

In Fig. 5, we show the death lines of the pulsars with outer gaps for two possible distributions

of the magnetic inclination angles. In this case, we assume that X-rays are produced by the neutron star cooling and polar cap heating. Because the temperature used by Hibsman & Arons (2001) is higher than that used by Zhang & Harding (2000), the average energy of X-rays for the former is greater than that for the latter. Therefore, the death lines obtained by using the temperature of Hibsman & Arons (2001) are lower than those by using the temperature of Zhang & Harding (2000).

In Fig. 6, we show the death lines of the pulsars with outer gaps, in which X-rays are produced by the returning relativistic particles from the outer gap, and the outer gap is self-sustained. For comparison, we also plot the death lines given by Chen & Ruderman (1993), and the death line derived from the model of Zhang & Cheng (1997). It can be seen that our model predicts that much more radio pulsars have self-sustained outer gaps compared to those given by Chen & Ruderman (1993) as well as by Zhang & Cheng (1997). In our estimate of the death lines of the pulsars with self-sustained outer gaps, we require that $f(r_{in}) = 1$. This condition is reasonable. It is believed that an outer gap can develop along the null charge surface (Cheng, Ruderman & Sutherland 1976) or along the last closed field line (CHR I; CHR II). Therefore, a self-sustained outer gap exists if the fractional size of the outer gap at r_{in} is not greater than unity.

5. Conclusion and Discussion

After taking the geometry of the dipole magnetic field, we have given a revised version of the outer gap model given by Zhang & Cheng (1997). In the revised outer gap model, the fractional size of the outer gap is not only the function of period and magnetic field of the neutron star, but also the function of the radial distance (r) to the neutron star and the magnetic inclination angle (α). In other words, the fractional size of the outer gap has a form of $f(r, \alpha) = f_i(P, B_{12})G_i(r, \alpha)$, where $f_0(P, B_{12})$ is only the function of pulsar period and surface magnetic field, $G_i(r, \alpha)$ changes with radial distance to the neutron star and the magnetic inclination angle and subscript i represents the X-ray field considered. The fractional size of the outer gap is given by Eq. (32) in the X-ray

field which is produced by the neutron star cooling and polar cap heating and by Eq. (36) in the X-ray field which is produced by outer gap heating. In this model, the fractional size of the outer gap has a minimum at the inner boundary for a given pulsar, increases with the radial distance along the last open field lines and then reaches its outer boundary where $f(r_b, \alpha) = 1$. In other words, the outer gaps of some pulsars do not start from the null charge surface to the light cylinder. We have shown the changes of the fractional size of the outer gap with the

inclination angle (see Fig.1). Further, we have found that the outer gaps of relative young pulsars such Vela can extend from the null charge surface to the light cylinder for any inclination angle, however, the outer gaps of some pulsars like as Geminga cannot extend to the light cylinder for a larger inclination angle (say 75°).

In order to describe the average properties of high-energy radiation from a γ -ray pulsar, we have defined an average radial distance $\langle r \rangle$ (see Eq. (38)). Using Monte Carlo method described by Cheng & Zhang (1998) (also see Zhang, Zhang & Cheng 2000), we simulated two populations of γ -ray pulsars whose energy fluxes are greater than EGRET threshold and GLAST threshold. In these simulations, we only consider the case of pulsars with self-sustained outer gap and uniform distribution of the inclination angles. We have plotted the change of L_γ with L_{sd} (see Fig. 2). We also indicated the variation of $L_\gamma/L_{\gamma,0}$ with L_γ in Fig. 3, which show the importance of the inclination angle on L_γ . In the model of Zhang & Cheng (1997), L_γ is independent on the inclination angle. Fitting the simulated results, we found that $L_\gamma \propto L_{sd}^{0.38}$ for EGRET threshold and $L_\gamma \propto L_{sd}^{0.46}$ for GLAST threshold. Compared with the observed data given by Thompson (2001), our simulated results are reasonable (see Fig. 4). In fact, current distance of the Vela pulsar (Cavareo, et al. 2001) is less than that used by Thompson (2001) about a factor of two, which reduce the luminosity about a factor of four.

In Fig.4, we note that Geminga is not within the simulated population. Zhang & Cheng (2001) have used a three-dimension outer gap model to explain the phase-resolved spectra of Geminga. They discovered that in order to fit the observed γ -ray data the solid angle $\Delta\Omega$ must be near $5sr$.

In Fig. 4, $\Delta\Omega = 1sr$ is used for all observed γ -ray pulsars. If the larger solid angle is used, it brings Geminga within the simulated population. It should be pointed out that Yadigaroglu & Romani (1995) have studied the effect of γ -ray beaming in their outer gap model (also see Romani 1996). Zhang et al (2000) also gave a approximate expression of the γ -ray beaming fraction, which will be applied to estimate γ -ray fluxes in our new model.

It is interesting to point out that both polar gap models and outer gap models predict $L_\gamma = L_{sd}$ for low spin-down power pulsars. But the position of the break occurs at $5 \times 10^{33} P^{-1/2} \text{erg/s}$ for the polar gap model (Harding et al. 2002) whereas the outer gap models predict a higher position at $1.5 \times 10^{34} P^{1/3} \text{ergs}^{-1}$. For higher spin-down power pulsars, the polar gap models predict $L_\gamma = L_{sd}^{1/2}$, whereas the outer gap cannot give precise prediction because the model L_γ also depends on a less well-known parameter α . The statistical predictions of the outer gap model give $L_\gamma = L_{sd}^\delta$, where δ depends on the properties of γ -ray detector. For example, $\delta = 0.38$ for the EGRET and $\delta = 0.46$ for GLAST.

According to our model, the fractional size of outer gap at the null charge surface for a given pulsar ($f(r_{in}, \alpha)$) reaches a minimum. Therefore, the outer gap should exist only if $f(r_{in}, \alpha) \leq 1$. Averaging $f(r_{in}, \alpha)$ on two possible (uniform and cosine) distributions of the magnetic inclination angles respectively, we have obtained the death lines of the pulsars with outer gaps in the two possible X-ray fields and the comparison them with the observed data in Figs.5 and 6.

Compared with the death line derived from the outer gap model of Zhang & Cheng (1997), the revised model predict that more pulsars will have their outer gaps and then emit high-energy photons.

We would like to make the conclusion as follows. Our results indicate that (i) the intrinsic parameters for explaining the observed γ -ray properties of rotation-powered pulsars are the magnetic inclination angle, period and magnetic field. We have obtained a very concrete functional form of the prediction of gamma-ray luminosity which only depends on these intrinsic parameters, the inclination angle could be known if the radio data is

sufficiently good; (ii) the conversion efficiency for 7 known gamma-ray pulsars, that is rather scattered, can be explained in our revised model using the Monte Carlo method. Although our estimation of the outer gap size cannot precisely predict the thickness of individual pulsar when the magnetic inclination angle is poorly known, it is a reasonable estimation of the statistical properties of gamma-ray pulsars using the statistical method; (iii) Unlike those 7 observed gamma-ray pulsars, mature pulsars (ages 0.3-3 million years) can also be gamma-ray pulsars and their efficiency is insensitive to the inclination angle. Most importantly, their gamma-ray luminosity is proportional to the spin-down power, which can be tested by GLAST; (iv) The mean cut-off age of gamma-ray pulsars is increased by a factor of 3, therefore older gamma-ray pulsars (up to about 3 million years old) can move up to higher galactic latitude. Some unidentified EGRET gamma-ray sources could be mature pulsars with ages between 0.3-3 million years old pulsars predicted by this model. In fact, Parkes Observatory survey has discovered a large number of radio pulsars on the error boxes of EGRET unidentified gamma-ray point sources (Torres et al. 2003); and (v) The previous works on death line based on the outer gap model (e.g. Chen and Ruderman 1993) did not give a detail physical reason rather they used a phenomenological approach. Here we proposed a new criterion based on concrete physical reason for the death line, which predicts more gamma-ray pulsars. (vi) Our model predicts many more weak γ -ray pulsars with age between 0.3 - 3 million years old than previous outer gap models and they satisfy $L_\gamma \propto L_{sd}$, which can be verified by GLAST.

Finally, we would like to remark that the gamma-ray luminosity formulae developed in this paper may not be able to explain the gamma-ray luminosity of individual pulsar. Two important factors, i.e. distance uncertainty and beaming fraction, which play crucial roles in determining the gamma-ray luminosity, have not been considered here. For example, the luminosity ratio between PSR B1055-52 and Geminga, which have same spin-down power of $3 \times 10^{34} \text{ erg s}^{-1}$, is about a factor of 6 (Thompson et al. 2001). But the model prediction (cf. Fig. 2) is no more than a factor of 3. In fact, the estimate of the distance of PSR B1055-52 is very uncertain, its

distance can change from ~ 1.5 kpc (for example, Thompson et al. 2001) to a small value of ~ 500 pc (see, for example, Ögelman & Finley 1993; Combi et al. 1997; Torres, Butt, Camilo 2001; Hirotani & Shibata 2002), which makes the ratio change from ~ 10 to ~ 3 . Recent distance estimate for PSR B1055-52 from the dispersion measure is ~ 0.72 kpc (see http://rsd-www.nrl.navy.mil/7213/lazio/ne_model, also see Mignani, DeLuca & Caraveo 2003). However, it is well known that 25% error is common in determining the dispersion measure (MaLaughlin & Cordes 2000), which gives a factor of 2 uncertainties in the luminosity. Also the distance estimate of Geminga is known to have at least 25% uncertainties (Caraveo et al. 1996), which give another factor of 2 uncertainties in gamma-ray luminosity estimate. Other uncertainty results from the gamma-ray beaming fraction. In principle, both distance estimate and beaming fraction will be obtained more accurate. Then our model still predicts that the difference in the magnetic inclination angle can still cause a large difference in gamma-ray luminosity for pulsars with same spin-down power.

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Fig. 1.— Variation of the fractional size of the outer gap with the magnetic inclination angle for some typical pulsar parameters. Panel (A): $\eta(P, B, \alpha)$ versus α for a given magnetic field of 3×10^{12} G. The upper (lower) solid line is the self-consistent outer gap model (the cooling X-ray outer gap model) with $P = 0.1s$ and the upper (lower) dashed line is the self-consistent outer gap model (the cooling X-ray outer gap model) with $P = 0.3s$ respectively. Panel (B): $\eta(P, B, \alpha)$ versus α for a given period of 0.2 s. The upper (lower) solid line is the self-consistent outer gap model (the cooling X-ray outer gap model) with $B_{12} = 3.0$ and the upper (lower) dashed line is the self-consistent outer gap model (the cooling X-ray outer gap model) with $B_{12} = 1.5$ respectively.

Fig. 2.— The change of γ -ray luminosity (L_γ) with the spin-down power (L_{sd}) in the γ -ray pulsar population predicted by our outer gap model. In our simulation, we have used the EGRET threshold as the minimum detectable γ -ray energy flux. Open circles and shaped circles are the model radio-quiet and radio-loud γ -ray pulsars respectively and the solid line is the best fit for all γ -ray pulsars with outer gaps. Shaded line is the best fit for the radio-loud γ -ray pulsars with outer gaps.

Fig. 3.— $L_\gamma/L_{\gamma,0}$ versus L_γ in the γ -ray pulsar population predicted by our outer gap model. In our simulation, we have used the EGRET threshold as the minimum detectable γ -ray energy flux. Open circles are the expected data and solid line is the best fit. For comparison, we show the result given by Zhang & Cheng (1997) as a dashed line.

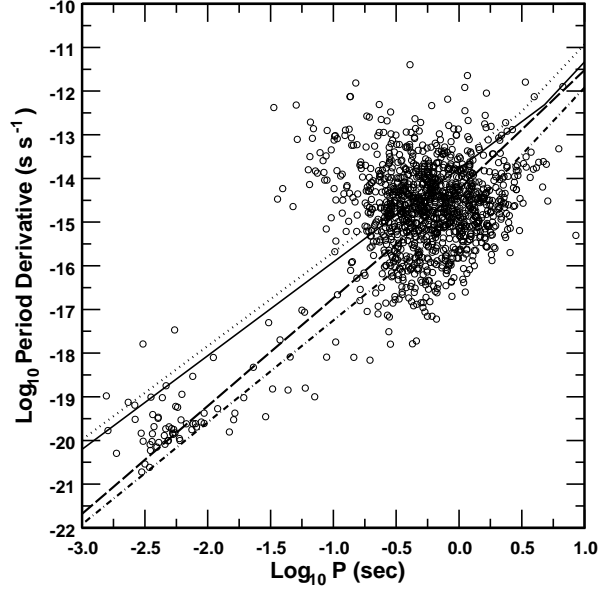


Fig. 5.— Death lines of the pulsars with outer gaps. It is assumed that X-rays are produced by the neutron star cooling and polar cap heating. Two cases of the inclination angle distributions are considered: (i) an uniform distribution and (ii) a cosine distribution. Dotted and solid lines are presented the death lines given by Eqs. (56) and (59). Dashed and dot-dashed lines represents the death lines given by Eqs. (57) and (60). The observed data are taken from see website <http://www.atnf.csiro.au/research/catalogue/>

Fig. 4.— γ -ray luminosity versus the spin-down power. Solid circles with error bar are the observed data given by Thompson et al. (2001), solid and dashed lines are our results given by Eqs. (50) and (51) respectively.

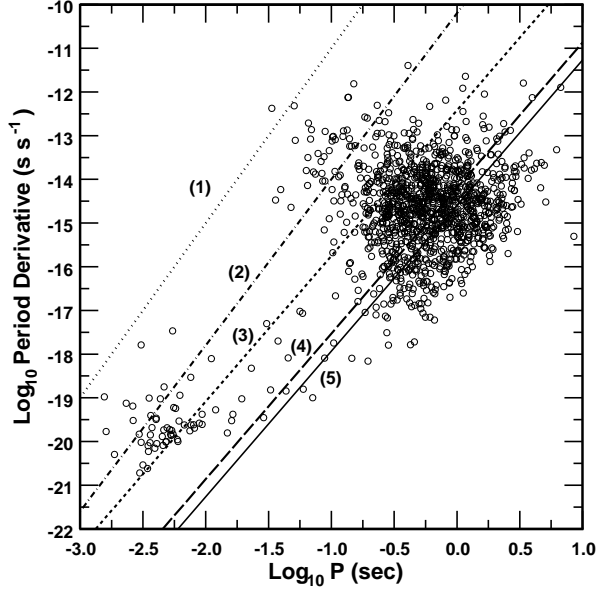


Fig. 6.— Death lines of the pulsars with self-sustained outer gaps. The observed data are taken from see website <http://www.atnf.csiro.au/research/catalogue/>. Lines (1) and (2) are given by Eqs. (62) and (63) respectively (Chen & Ruderman 1993). Line (3) is the death line (Eq. (64)) predicted by Zhang & Cheng (1997). Lines (4) and (5) are the death lines of our model for the uniform and the cosine distributions of the inclination angles, respectively.

